Supplementary note: an example illustrating the approximation used for the likelihood

An example motivating this approximation

To motivate the approximation in the likelihood (section B1.2), we consider a simple, univariate example where the output of the numerical solver is 1000 for three consecutive time steps, and both the measurement error variance and numerical error variances are 1 – note that the magnitude of the numerical solver is a few orders of magnitude larger than the measurement error. That is, we have:

$$
\begin{aligned}
Y_1 &= 1000 + X_1 + Z_1 \\
Y_2 &= 1000 + X_1 + \epsilon_1 + Z_2 \\
Y_3 &= 1000 + X_1 + \epsilon_1 + \epsilon_2 + Z_3
\end{aligned}
$$

Where Z_1, Z_2, Z_3, X_1, ϵ_1 ,and ϵ_2 are all identically and independently distributed $N(0,1)$ (normal with 0 mean and unit variance) random variables. Analytically, the conditional distribution of $p(Y_3|Y_2,Y_1)$ follows a normal distribution with mean $1000 + .2(Y_1 - 1000) + .6(Y_2 - 1000)$ and variance 13/5. In our approximation, we substitute this distribution with a $N(Y_2, 3)$; to show that these distributions are indeed quite close to each other, we conduct 25 simulations and illustrate $P(Y_3|Y_2,Y_1)$ in Figure 1. These results motivate the use of the approximations in section B1.2.

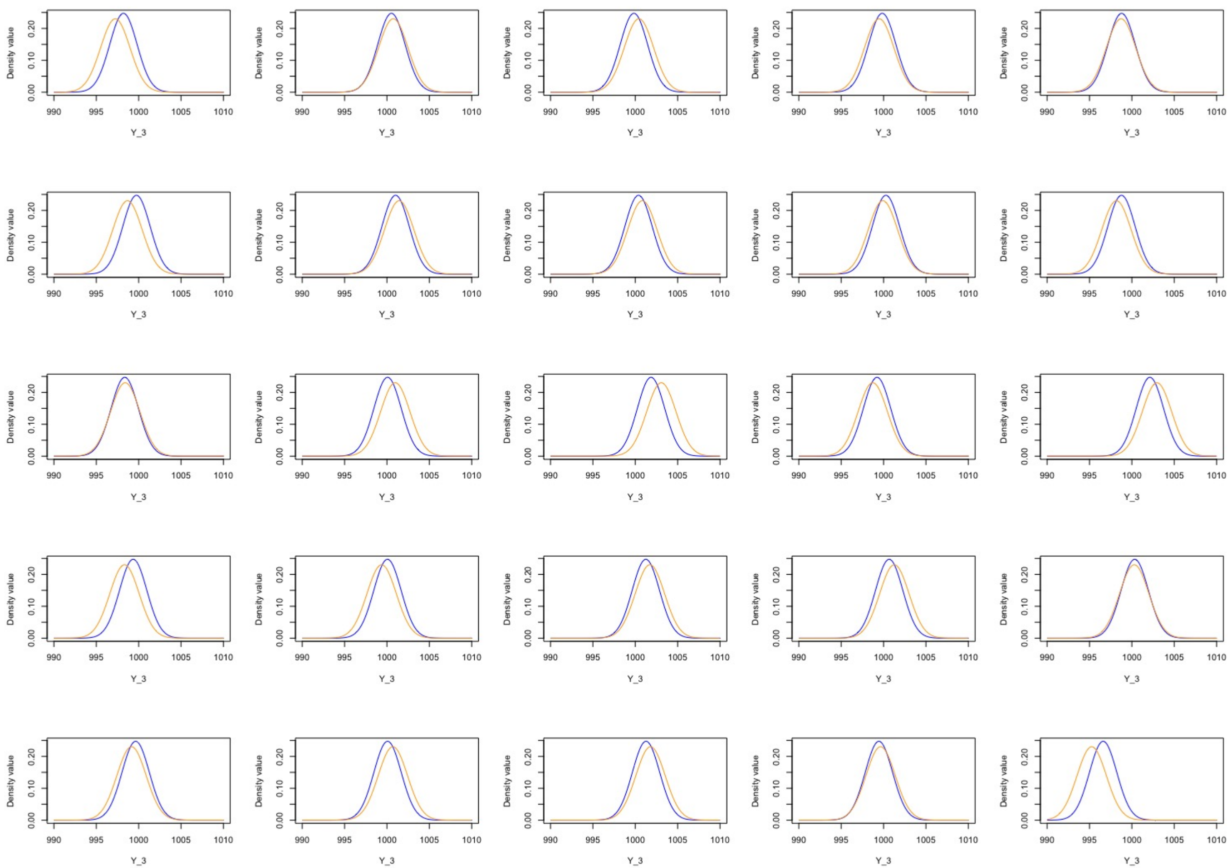

Figure 1: 25 simulations from the above model with $P(Y_3|Y_2, Y_1)$ in blue and $N(Y_2, 3)$ in orange. Visual inspection of these densities in all of the simulations shows that they are close to each other, lending evidence that it is appropriate to use our approximation in the regime where the output of the numerical solver is much larger than measurement errors.