MISMIP: Marine ice sheet model intercomparison project

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1 Introduction

The last few years have seen a great deal of effort invested by various research groups in developing numerical marine ice sheet models that are intended to represent accurately the motion of an ice sheet grounding line. This effort has seen a variety of models emerge. It is at present unclear to what extent these models agree with one another, or indeed, how well they are able to model real marine ice sheets.

The purpose of the proposed experiments is to address the former question: how well do our models agree with one another? In addition, the purpose of the experiments is also to assess how well the numerical schemes used solve the partial differential equations on which they are based. Not all the models used in marine ice sheet simulations deal with the same equations, but many of them share basic characteristics. For instance, most include a representation of the longitudinal stresses required to couple ice shelves to the ice sheet. We therefore do not restrict this intercomparison to a single type of model, but invite a variety of models described below to participate.

The impetus for the intercomparison has been provided amongst others by the recent papers of *Vieli and Payne* (2005), *Pattyn et al.* (2006), *Hindmarsh* (2006) and *Schoof* (2007a,b), who have demonstrated not only the strong possibility of numerical artifacts in marine ice sheet simulations, but also the importance of grid resolution and of accurate represention the sheet-shelf transition zone. These papers have also shed some light on the question of ice sheet stability. No stable steady states have been found on upward-sloping ice sheet beds, in line with an early hypothesis of *Weertman* (1974). However, some of the numerical results in *Vieli and Payne* (2005) and *Pattyn et al.* (2006) have left open the possibility of 'neutral equilibrium', meaning that a perturbation in grounding line position in a steady-state ice sheet could lead to a similar, but distinct, new equilibrium shape. This notion has, however, been discounted by *Schoof* (2007b), who also found that marine ice sheets may undergo dramatic and irreversible changes — hysteresis — under changes in physical properties (sliding, ice viscosity) or external forcing (accumulation rates, sea levels). The basic questions that arise from these papers and earlier work are the following:

- Do marine ice sheets have one or more distinct equilibrium shapes (*Weertman*, 1974; *Thomas and Bentley*, 1978), or do they exhibit 'neutral equilibrium' (*Hindmarsh*, 1996): will a perturbation in grounding line position away from steady state result in the grounding line either returning to the (stable) steady state position or migrating away from the (unstable) steady state position to a stable steady state?

- Do stable steady states have to have their grounding lines in a region of downward-sloping bed?

- How do equilibria depend on bed geometry and the physics of sliding at the bed, ice viscosity and gravitational forces as well accumulation rates?

- Is hysteresis under changes in forcings and internal physical properties possible when the bed is overdeepened?

- To what extent is high grid resolution, especially near the grounding line, necessary to obtain reliable results? Is this particularly important when modelling transients?

To facilitate comparison, we will use the simplest physical set-up that can represent a marine ice sheet:

- The ice sheet is two-dimensional, so there is no lateral shearing or buttressing, and no varying flowline width, and the ice sheet is further symmetrical about an ice divide.

- Sliding is described by a power law linking basal shear stress τ_b to sliding velocity u_b: $\tau_b = C|u_b|^{m-1}u_b$, with C and m constant

- Glen's law is used with constant coefficients A and n

- Longitudinal stresses couple the ice sheet to the ice shelf at the grounding line, where the ice becomes afloat, such that the longitudinal stress is continuous across the grounding line.

We invite all marine ice sheet models that can be adapted to include these physical assumptions in some form to participate.

One recent development that will hopefully facilitate the intercomparison is the development of a boundary layer theory for sheet-shelf interactions (*Schoof*, 2007a). This theory takes a complementary approach to numerical marine ice sheet models: it uses a systematic set of approximations to parameterize the sheet-shelf transition zone in a simpler, shallow ice model. The difference with other models is that the theory is not specific to a particular numerical method. Importantly, it allows steady states to be computed semi-analytically and at low computational cost, and we will use these steady states to guide the experimental design. We emphasize, however, that the boundary layer theory is itself only approximate, and part of the objective of the experiment is to test it against other models.

The remainder of this document is mostly intended as a technical description of the experiments that we are proposing.

2 Experiments

Before we describe the experiments, a note on our philosophy: If you are unable to conduct all experiments described below, partial results are permissible, but we encourage results from the full suite of experiments to facilitate comparison. Of course, this may be difficult for the more computationally expensive models that explicitly include vertical shearing, such as Blatter's model. If the computational demands of your model are too high to conduct all the transient experiments below, we welcome submissions that provide only steady-state results. Please contact the organizers in advance if in doubt.

The basic aim of the experiments described below is four-fold:

- At the simplest level, we will establish whether all models relax to the same equilibria, and whether there are differences in equilibrium shapes depending on whether they are approached in advance or retreat.
- We will compare the transient approach to steady state: do the models take the same time to relax, and do the same initial conditions give rise to the same steady states?
- We will investigate the effect of grid resolution on results
- Does an overdeepened bed lead to hysteretic behaviour because steady, stable grounding line positions cannot be located where the bed is upward-sloping?

We describe the experiments in detail next, and then define grid modes and the types of participating models that are envisaged.

2.1 Experiment 1: Relaxation to steady state on a downward-sloping bed

The theory in *Weertman* (1974) suggests that, for a fixed accumulation rate, there should be single stable equilibrium profile for a marine ice sheet on a downward sloping bed. To compare model output against this result, we will use the following experiment: We use a simple bed shape with a constant downward slope given by

$$b(x) = -\left(720 - 778.5 \times \frac{x}{750\ \text{km}}\right)\ \text{m} \tag{1}$$

as in *Schoof* (2007b), figure 9. (Note that notation for bed geometry in MISMIP has b measured *downwards* from sea level, asdisplayed in figure 1.) In order to attain different equilibria, we vary the Glen's law parameter A as shown in table 2: larger A should correspond to smaller ice sheets. There are other means of changing equilibria, for instance changes in bed slipperiness C, accumulation rate a, or changes in sea level (*Schoof*, 2007b). Here we choose changes in A (corresponding to changes in ice temperature) because they are easy to implement. Semi-analytic results for grounding line position as a function of A based on boundary layer theory (model B1 in section 4.5 below) figure are shown in figure 2, and we aim to compare model results against this figure. Table 1 at the end of

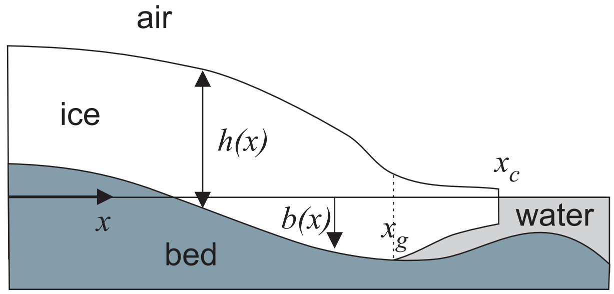

Figure 1: Geometry of the ice-sheet set-up.

the document lists the symbols used for various parameters that we are prescribing, and the numerical values to be used for these parameters are given in tables 2–4 (also at the end of the document). To gain a better understanding of the physical meaning of these parameters, consult section 4.1

We also use the fixed parameter values for accumulation rates a, ice and water densities ρ_i and ρ_w, acceleration due to gravity g and Glen's law exponent n in table 3. The experiment will be conducted with two different sets of sliding parameterizations, using values of C and m as shown in table 4. The two versions of the experiment will be labelled experiments 1a and 1b. The rationale for this is that the results in *Schoof* (2007b) only pertain to the case $m = 1/n = 1/3$, while many other marine ice sheet models have used a linear sliding law with $m = 1$. For both sliding law exponents ($m = 1/3$ and $m = 1$), the coefficient C in table 4 is chosen to give sliding velocities around 35 m year^{-1} for a driving stress of 80 kPa.

Each model run (with the values of A and C, m as described above) is to be conducted as follows: For step # 1 in table 4, start with a ten metre layer of ice on land, extended up to the position where this ten metre layer becomes afloat ($x = 702.3$ km). Optionally, a ten metre thick ice shelf can be attached. The model is then to be run until a steady state is attained. As a termination criterion for having achieved a steady state, we require that the rate of change of grounding line position be $.1$ m year^{-1} or less, while the rate of change of ice thickness at each grid point at which ice thickness is defined must be less than 1^{-4} m year^{-1}. Failing that, model runs can be terminated after 3×10^4 years. For step # 2, start with the final profile in step # 1 and run the model to equilibrium, using the same termination criteria as for step # 1. For step # 3, repeat the procedure, using the final profile from step # 2, and so on.

Some early submissions have shown that, for experiment 1b, the final steady state in step # 9 for some models may lie outside the 1800 km domain size specified for models with fixed grids in section 2.4. If this happens in your runs, terminate experiment 1b after step # 8, and use this as the initial shape for experiment 2b below.

N.B. These specifications have been changed from the original set-up of experiment 1 (draft dated 25 September 2007, which required a 10 m thick ice sheet at the start of each run. The reason for the alterations is that preliminary results from models with an ice shelf have indicated that this specification of initial conditions leads to ice shelf thickness becoming very small in the early stages of the ice sheet growth,

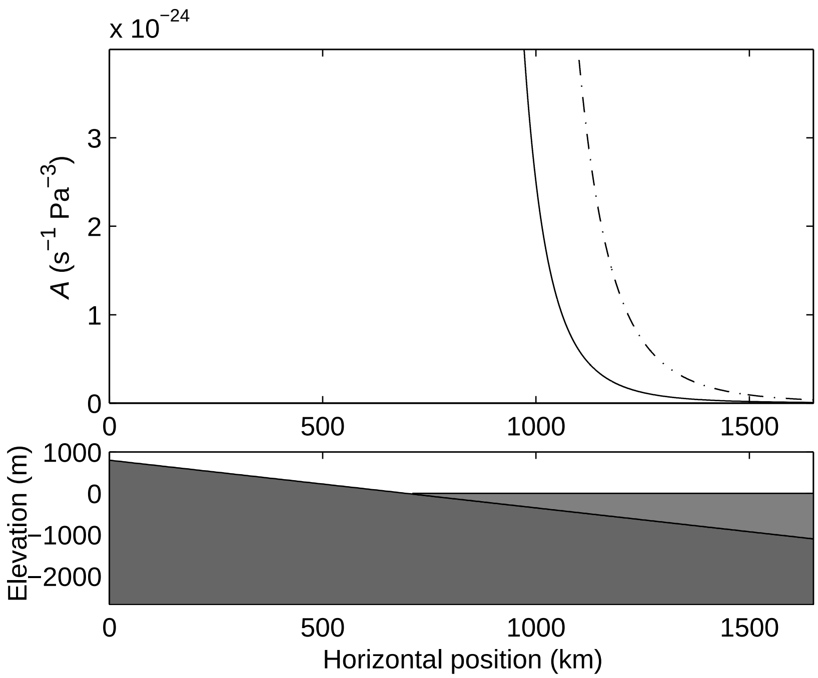

Figure 2: Values of the Glen's law parameter A corresponding to steady grounding line positions for the downward-sloping bed given by (1), as predicted by the boundary layer theory in *Schoof* (2007a). The dash-dotted line corresponds to experiment 1b, the solid line to experiment 1a. The bottom panel shows the shape of the bed, with the bed in dark grey and the ocean in light grey.

which leads to severe numerical difficulties. If your model encounters this problem in run # 1, then use a steady-state profile calculated numerically for model B1 below with the parameter values for run # 1, and a corresponding steady-state shelf profile based on the category 2 model below. The script `SMsolver2.m` **in the MATLAB package** `MISMIP_distribution.tar` **available from the MISMIP website will produce these steady state profiles on a user-defined grid. Type** `help SMsolver2` **in MATLAB for further details on how to use this script and the other functions in the package, and contact Christian Schoof for further assistance if necessary.**

2.1.1 Required output for Experiment 1

We require the following for each run:

- At 50 year intervals, provide grounding line position $x_g(t)$, grounded ice volume $\int_0^{x_g} h \, \mathrm{d}x$ and ice thickness at the ice divide, $h(0, t)$, to assess transient behaviour

- If your model employs a formula for grounding line motion based on differentiating the flotation condition (8), i.e.

$$\frac{\mathrm{d}x_g}{\mathrm{d}t} = -\frac{a - \frac{\partial q}{\partial x}}{\frac{\partial h}{\partial x} + \frac{\rho_w}{\rho}\frac{\partial b}{\partial x}}, \tag{2}$$

  then also provide ice thickness $h(x_g, t)$ at the same 50 year intervals as $x_g(t)$ etc., as described above.

- If your model does not use an explicit formula of this type, then we require data to evaluate

$$\frac{\mathrm{d}x_g}{\mathrm{d}t} - \frac{a - \frac{\partial q}{\partial x}}{\frac{\partial h}{\partial x} - \frac{\rho_w}{\rho}\frac{\partial b}{\partial x}} \tag{3}$$

  at the grounding line at the same 50 year intervals as $x_g(t)$. The output required at these time intervals is: positions of two grid points immediately to the grounded side of the grounding line (one of which can be the grounding line itself), plus ice flux q, ice thickness h and depth-to-bed b at these locations. If your model resolves the ice shelf, provide the same information for one grid point immediately to the floating side of the grounding line (this point must be distinct from the grounding line).

  If your model makes it difficult to compute an ice flux ($q = \int_{-b}^{h-b} u \, \mathrm{d}z$), please contact the organizers before preparing your results.

- Further, provide the full final equilibrium profile as grid point positions x_i, grounding line position x_g, and ice thicknesses $h(x_i, t_{\text{final}})$, and the termination time t_{final}. Also provide a note explaining whether termination occured because the steady-state tolerances described above were satisfied, or because a time of 30 kyr was reached.

- We also recommend that participants store the full ice sheet profile at the 50 year intervals specified above on their own computer systems. This is to facilitate additional checks if any queries arise. However, *do not* upload these data unless requested.

2.2 Experiment 2: Reversal of parameter changes

The discussion about neutral equilibria in marine ice sheets makes (*Hindmarsh*, 1996) motivates experiment 2, which is a follow-up to experiment 1. We wish to establish whether the ice sheet relaxes to the same equilibrium profile under retreat as under advance.

Start experiment 2 with the final ice sheet profile obtained from experiment 1. With this initial condition, set A to the value in run no. 8 in table 4, and allow the ice sheet to relax to a new equilibrium. Use the same criterion for having achieved equilibrium as in experiment 1. Then reduce A to the value in run no. 7 in table 4, and again allow the ice sheet to relax to equilibrium. Proceed in this fashion until you reach the value of A in run no. 1 in table 4. As in experiment 1, we distinguish between experiment 2a and 2b depending which combination of m and C values are used in table 3.

2.2.1 Required output for Experiment 2

For each 'run' (value of A in table 4), report the final equilibrium profile as grid point positions x_i, grounding line position x_g and $h(x_i, t_{\text{final}})$, as in the penultimate bullet point in section 2.1.1.

2.3 Experiment 3: Hysteresis

The results in *Schoof* (2007b) indicate that ice sheets can undergo hysteresis under parameter variations when the shape of the bed has an overdeepening. This experiment aims asses whether other ice sheet models exhibit the same behaviour, and to assess further how transients differ between different models and discretizations. We deliberately use step changes in parameters to isolate the threshold behaviour expected (and to allow comparison with semi-analytical steady states by allowing relaxation to steady state following each step change). We also anticipate that different models will have different thresholds and that model results in this experiment may differ significantly. This would be an important observation as it indicates that observations of thresholding behaviour in numerical simulations of real ice sheets may be subject to significant uncertainty.

We use the same overdeepened polynomial bed shape as in *Schoof* (2007b):

$$b(x) = -\left[729 - 2184.8\times\left(\frac{x}{750\text{ km}}\right)^2 + 1031.72\times\left(\frac{x}{750\text{ km}}\right)^4 - 151.72\times\left(\frac{x}{750\text{ km}}\right)^6\right] \text{ m.} \tag{4}$$

Parameter choices for a, n, ρ_i, ρ_w and g are again those in table 3. We again conduct the experiment with the two sets of parameter choices for C and m given in table 4, and label these sub-experiments 3a and 3b. The grounding line positions predicted by the category 0 model A1 (see section 4.5) are shown in figures 3 and 4, and we aim to establish whether other models reproduce these curves.

The values for A to be used in experiments 3a and 3b are given in tables 5 and 6. Each sub-experiment, 3a and 3b, will be conducted as follows: As in experiment 1, set up the model domain with a 10 m thick grounded ice layer up to the location where this becomes afloat ($x = 482.4$ km), and with an optional attached 1 m thick ice shelf. Then run the model with the value A given for 'step 1' in table 5 or 6, as appropriate, for the time interval given for 'step 1'. Then change the value of A to that given for 'step 2' but leave the ice sheet geometry unchanged. Continue running the model for the time interval given for 'step 2'. Continue in this fashion until you reach the end of the table. (Note: the different time intervals have been chosen to account for the fact that some changes in A will result in irreversible transitions across the overdeepening, and these transitions can be anticipated to take longer to complete than the smaller ice sheet adjustments that occur for other steps in A, and we base our anticipated thresholds on model B1 in section 4.5 below. Generally, the anticipated threshold vales of A at which these transitions are expected have a 'safety bracket' so that other models, which may have somewhat different thresholds, can also relax to steady state.).

N.B. As in experiment 1, preliminary results from models with an ice shelf have indicated that the specification of initial conditions with uniform ice thickness of 10 m in sheet and shelf (step 1) can lead to ice shelf thickness becoming very small in the early stages of the ice sheet growth, which leads to severe numerical difficulties. If your model encounters this problem in run # 1, then use a steady-state profile calculated numerically for model B1 below with the parameter values for run # 1, and a corresponding steady-state shelf profile based on the category 2 model below. The script `SMsolver2.m` **in the MATLAB package** `MISMIP_distribution.tar` **available from the MISMIP website will produce these steady state profiles on a user-defined grid. Type** `help SMsolver2` **in MATLAB for further details on how to use this script and the other functions in the package, and contact Christian Schoof for further assistance if necessary. Note that the package as distributed has depth to bed b and bed slope $\partial b/\partial x$ set up for experiment 1 (i.e., based on equation (1)). To change these to equation (4), you will need to comment out and uncomment the relevant lines in the MATLAB functions** `SMcold_bedheight.m` **and** `SMcold_bedslope.m`.

2.3.1 Required output for Experiment 3

We require the same output as for experiment 1 (see section 2.1.1). The 'equilibrium profiles' in last bullet in section 2.1.1 are now simply the profiles at the end of each time interval in the various steps in tables 5 and 6.

2.4 Grid spacing and time steps

Each of the experiments 1 and 2 below will be conducted at three grid resolutions:

- Mode 1: Low resolution. If your model uses a fixed grid, use a domain ranging from $x = 0$ to $x = 1800$ km, and use 150 equally spaced horizontal grid points in your computational domain. If using a moving grid (employing the spatial coordinate $\sigma = x/x_g$), use 100 equally spaced horizontal grid points in the part of your computational domain occupied by the grounded sheet (i.e., 100 equally spaced grid points between $\sigma = 0$ and $\sigma = 1$). In both cases, if using a staggered grid, use 150 (100) equally spaced points on each grid: i.e., use 150 (100) grid points for thickness h and 150 (100) grid points for for velocity u. If using a higher-order or other model that includes a vertical (z-) coordinate, you may use as many vertical layers as you

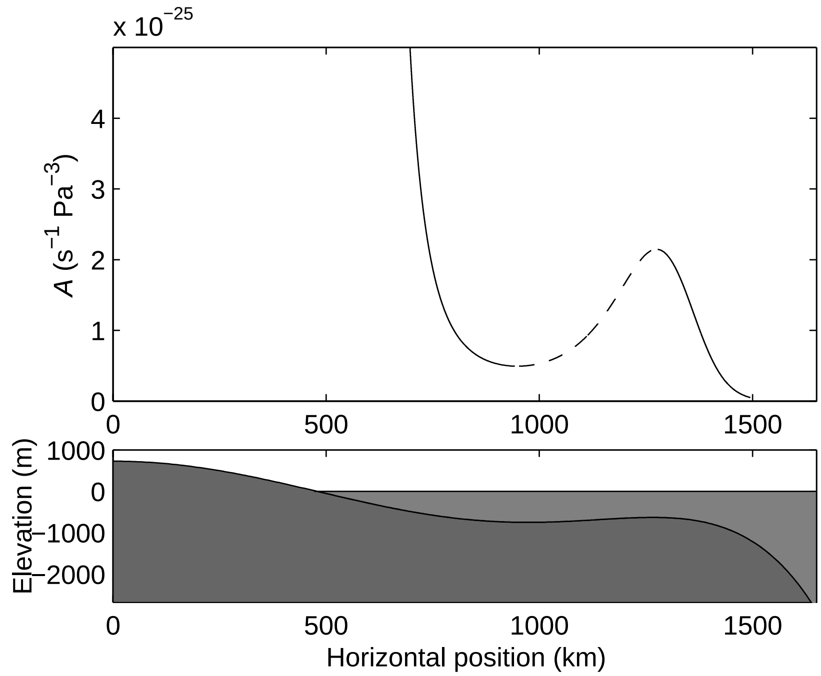

Figure 3: Steady-state grounding line position for the overdeepened bed given by equation 4 (shown in the bottom panel) as a function of A for experiment 3a. The dashed line corresponds to unstable steady states that are not expected to be observed in practice.

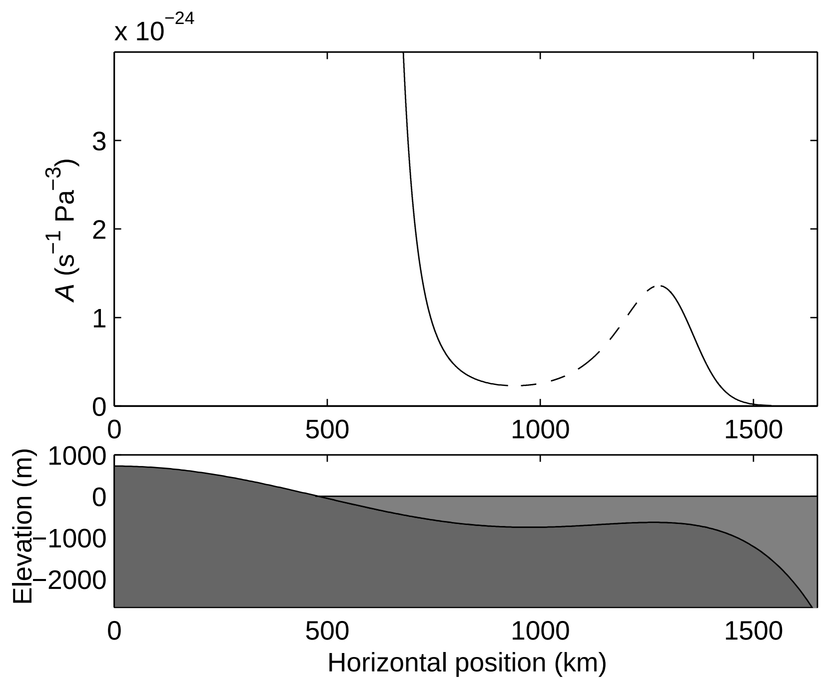

Figure 4: Steady-state grounding line position for the overdeepened bed given by equation 4 as a function of A for experiment 3b.

wish (though you must state how many layers are used when submitting results). Do not use nested grids or local grid refinements in this mode.

- Mode 2: As for mode 1, but use 1500 equally spaced horizontal grid points in the domain if using a fixed grid, and 1000 equally spaced horizontal grid points if using a moving grid.
- Mode 3: Define your grid at will. This mode allows for local grid refinements near the grounding line, which may be crucial to obtain accurate results. The aim of this mode is to produce the most accurate results you can.

The different specifications for fixed and moving grids are intended to account for the fact that, with the same number of grid points, the effective grid spacing will be smaller for a moving grid when the ice sheet is small than for a fixed grid, which will incorporate many grid points lying outside the grounded part of the ice sheet.

To distinguish between these modes in each experiment, a suffix notation will be used: for instance, experiment 1a run in mode 1 will be experiment 1a-M1.

Time steps: these may be specified by the user as needed to satisfy stability criteria, and must be specified when uploading results. They should be chosen such that results are insensitive to changes in the time step.

3 Discretization

How a particular model is discretized is up to each modeller. The aim of the intercomparison exercise is to compare different ways of solving (and hence, discretizing) the models listed below. Specifically, manipulations of the equations described above are permitted (for instance, coordinate stretchings, or the differentiation of the flotation condition (8) to obtain an evolution equation for x_g). When submitting results, a full description of the discretization used must be supplied. Include electronic offprints/preprints of relevant papers, if available, or give references. We will categorize different discretization schemes as appropriate at the time of evaluating results.

4 Admissible Models

We will consider models for two-dimensional marine ice sheets, in which there is no lateral shearing or 'buttressing' of ice shelves. In the same vein, models must not include representations of the effect of varying flowline width (as is effectively also the case in axisymmetric marine ice sheet models). The models must also be run with a fixed rigid bed and fixed sea levels.

We describe the various admissible categories of model next. If in doubt about which category your model fits into or how it should be configured, please contact the organizers in advance of preparing your results.

4.1 The basic model: category 1

The basic model on which the model intercomparison is built is the simplest possible model for a marine ice sheet that can account for the longitudinal stresses that couple ice shelf to ice sheet; all the parameters listed in table 1 the previous section pertain to this model. It describes a rapidly sliding ice sheet in which there is no variation in ice velocity across the thickness of the ice (e.g. *Muszynski and Birchfield*, 1987). This is also the model for which a simple boundary layer theory is available (see section 4.5).

Let h be ice thickness and u ice velocity, while b is the depth of the ice sheet bed below sea level and x is horizontal position. The grounded ice sheet occupies the region $0 < x < x_g$, where the grounding line position x_g can change over time t. Then mass and momentum conservation are described by

$$\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} = a, \tag{5}$$

$$\frac{\partial}{\partial x}\left[2A^{-1/n}h\left|\frac{\partial u}{\partial x}\right|^{1/n-1}\frac{\partial u}{\partial x}\right] - C|u|^{m-1}u - \rho_i g h \frac{\partial (h-b)}{\partial x} = 0. \tag{6}$$

ρ_i and g are ice density and acceleration due to gravity, respectively, while A is the rheological coefficient in Glen's law, and n is the corresponding exponent. a is ice accumulation rate, while the term $C|u|^{m-1}u$ represents friction at the bed, assumed to behave as $\tau_b = C|u_b|^{m-1}u_b$. Note that we have defined b to be positive if the bed is below sea level, i.e., depth of the bed is measured downwards (see also figure 1).

The centre of the symmetrical ice sheet is located at $x = 0$. Symmetry implies that

$$\frac{\partial (h-b)}{\partial x} = u = 0 \qquad \text{at } x = 0. \tag{7}$$

At the grounding line position, $x = x_g$, the ice becomes afloat, and coupling with the ice shelf (assumed unbuttressed) imposes a longitudinal stress:

$$h = \frac{\rho_w}{\rho_i} b, \tag{8}$$

$$2A^{-1/n}h\left|\frac{\partial u}{\partial x}\right|^{1/n-1}\frac{\partial u}{\partial x} = \frac{1}{2}\rho_i\left(1 - \frac{\rho_i}{\rho_w}\right) g h^2 \qquad \text{at } x = x_g. \tag{9}$$

Here, ρ_w is the density of water.

All material model parameters like A, C, m, n, ρ_i, ρ_w and g will be treated as constants (independent of position and time). No lateral shearing or buttressing effects, or representations of varying flowline widths will be considered; if these are included in a particular code, they must be switched off. The model parameters to be used will be specified below.

4.2 Sheet with an attached shelf: category 2

The boundary conditions (8) and (9) arise from integrating equations for an attached ice shelf. An alternative formulation is to resolve the attached shelf. Let the shelf occupy a domain $x_g < x < x_c$, where x_c can also evolve if desired. The shelf must not make contact with the bed. Then mass and momentum conservation for the shelf are

$$\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} = a, \tag{10}$$

$$\frac{\partial}{\partial x}\left[2A^{-1/n}h\left|\frac{\partial u}{\partial x}\right|^{1/n-1}\frac{\partial u}{\partial x}\right] - \rho_i(1-\rho_i/\rho_w)gh\frac{\partial h}{\partial x} = 0, \tag{11}$$

while equations (5) and (6) are retained as descriptions for the grounded sheet.

The symmetry condition (7) is maintained at $x = 0$. At the grounding line, we assume no jumps in ice flux, ice thickness and longitudinal stress:

$$h,\ u \text{ and } \frac{\partial u}{\partial x} \text{ are continuous at } x = x_g, \text{ with } h = \frac{\rho_w}{\rho_i}b. \tag{12}$$

At the calving front, there is an imbalance between hydrostatic pressures in ice and water due to the buoyancy of ice. This imbalance must be compensated by a longitudinal stress (e.g. *Shumskiy and Krass*, 1976):

$$2A^{-1/n}h\left|\frac{\partial u}{\partial x}\right|^{1/n-1}\frac{\partial u}{\partial x} = \frac{1}{2}\rho_i g\left(1-\frac{\rho_i}{\rho_w}\right)h^2 \qquad \text{at } x = x_c. \tag{13}$$

x_c can either be held fixed or a calving relation can be specified. There must always be a shelf in this model formulation, i.e., calving must not remove the shelf entirely in the model runs so that $x_c > x_g$ at all times.

As before, model parameters such as A, C, m, n, ρ, ρ_w and g must be treated as constants.

4.3 Higher order and other vertical-shear-resolving models: category 3

Higher-order or shear resolving models (e.g., Blatter's higher-order model (*Blatter*, 1995), models that solve Stokes' equations, or models analogous to categories 1 and 2 but with a representation of vertical shearing in the ice) are also encouraged. These must include the following features:

- As above, use only one horizontal dimension x, and do not include effects such as varying flowline width
- Lateral shearing and buttressing will again not be considered
- Sliding must be described by a friction-velocity relationship of the form $\tau_b = C|u_b|^{m-1}u_b$
- Glen's law must be used
- The models must be run in an 'isothermal' mode, i.e., the Glen's law parameter A must be treated as constant with depth in the ice. This will be the case even when the value of A suggests subtemperate ice (in which case, in a realistic simulation, no sliding might be assumed).

- Longitudinal stresses must be included, but by contrast with categories 1 and 2, vertical shearing must also be included (whether the model is depth-integrated or not)
- By analogy with categories 1 and 2, an ice shelf may be included explicitly, or flotation and stress conditions analogous to (8) and (9) can be used
- A symmetry condition of the form (7) applies at the ice divide $x = 0$

Again, all material model parameters (A, C, m, n, ρ_i, ρ_w and g in tables 2–4) must be treated as constants (independent of position and time). A full description of the model equations used must be submitted along with results. If using a model in which normal stress at the bed is not cryostatic (for instance, a full Stokes equation model), ensure that you state explicitly how flotation at the grounding line is handled.

4.4 Other models: category 4

Other marine ice sheet models that do not fit into the categories above are also welcome to participate. This explicitly includes shallow-ice models that have a moving grounding line (other than the specific shallow ice models described in the next section, which are excluded from category 4). The following restrictions on model formulation are again required:

- Use only one horizontal coordinate x, and do not include the effect of varying flowline width etc.
- Do not include lateral shearing and buttressing
- Use a sliding law of the form $\tau_b = C|u_b|^{m-1}u_b$
- Use Glen's law with constant coefficient A
- Use constant material parameters such as A, C, m, n, ρ_i, ρ_w and g
- The ice sheet must be symmetrical about $x = 0$

4.5 Asymptotic models: category 0

In this section, we describe four shallow ice-type models that are simpler than those in categories 1–4. These shallow ice models are based on the boundary layer theory due to *Schoof* (2007a). As described in *Schoof* (2007b), they are in good agreement with numerical results for the category 1 model, at least for the particular discretizations employed in that paper. All the shallow ice models consider only the grounded part of the ice sheet $0 < x < x_g$, and apply a moving boundary condition at $x = x_g$ to evolve the grounding line position.

4.5.1 Model A1

This is model A of *Schoof* (2007b), and is intended to provide a good approximation to category 1 and 2 models.

The ice sheet interior is described by a shallow ice-type equation in which ice flux $q = uh$ arises purely through sliding:

$$q = -\left(\frac{\rho_i g}{C}\right)^{\frac{1}{m}} h^{1+\frac{1}{m}} \left|\frac{\partial(h-b)}{\partial x}\right|^{\frac{1}{m}-1} \frac{\partial(h-b)}{\partial x}. \tag{14}$$

The continuity equation for the ice sheet becomes

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = a. \tag{15}$$

The prescription for grounding line migration in terms of local bed and ice geometry at $x = x_g$ is:

$$\begin{aligned}\left(\frac{\rho_w}{\rho_i}\frac{\partial b}{\partial x} - \frac{\partial h}{\partial x}\right)\frac{\mathrm{d}x_g}{\mathrm{d}t} &= -\frac{A}{4^n}(\rho_i g)^n(1-\rho_i/\rho_w)^n h^{n+1} \\ +a + \left(\frac{\rho_i g}{C}\right)^{\frac{1}{m}} h^{\frac{1}{m}} &\left|\frac{\partial h}{\partial x} - \frac{\partial b}{\partial x}\right|^{\frac{1}{m}-1}\left(\frac{\partial h}{\partial x} - \frac{\partial b}{\partial x}\right)\frac{\partial h}{\partial x}\end{aligned} \tag{16}$$

This is combined with a flotation condition (8) at $x = x_g$:

$$h = \frac{\rho_w}{\rho_i} b. \tag{17}$$

At the ice divide, the symmetry condition (7) applies.

4.5.2 Model B1

This is model B of *Schoof* (2007b), and is similar in its scope and formulation to model A1. The only difference from model A1 is that the grounding line migration prescription (16) is replaced by a flux condition:

$$q(x_g) = \left(\frac{A(\rho_i g)^{n+1}(1-\rho_i/\rho_w)^n}{4^n C}\right)^{\frac{1}{m+1}} [h(x_g)]^{\frac{m+n+3}{m+1}}. \tag{18}$$

4.5.3 Model A3 and B3

The boundary layer theory in *Schoof* (2007a) can be extended to deal with ice sheets that also have some shearing across the ice (Schoof, paper in preparation). We will use this version of the boundary layer theory in order to facilitate benchmarking of category 3 model results, which allow for representations of vertical shear. The equivalent of models A1 and B1 in this case are the same as model A1 and B1, respectively, except that the flux prescription in (14) is altered to

$$q = -\frac{A(\rho_i g_i)^n}{n+2} h^{n+2} \left|\frac{\partial(h-b)}{\partial x}\right|^{n-1} \frac{\partial(h-b)}{\partial x} - \left(\frac{\rho_i g}{C}\right)^{\frac{1}{m}} h^{1+\frac{1}{m}} \left|\frac{\partial(h-b)}{\partial x}\right|^{\frac{1}{m}-1} \frac{\partial(h-b)}{\partial x}. \tag{19}$$

This can be seen to agree with (14) in the limit where the first flux term in (19) (due to shearing) is small compared with the second (due to sliding).

5 Timeline and upload procedures

5.1 Timeline

The project was launched in September, 2007. The deadline for submission of final results was September, 2009. If you wish to participate in MISMIP, please contact the organizers via e-mail at cschoof@eos.ubc.ca, rcah@bas.ac.uk and fpattyn@ulb.ac.be, and also include the ISMIP coordinator, Philippe Huybrechts, at phuybrec@vub.ac.be.

5.2 Web addresses and upload site

Details and updates for the project will continue to be posted at

`http://homepages.ulb.ac.be/~fpattyn/mismip`

while the ISMIP project homepage (which acts as an umbrella for MISMIP and currently three other intercomparisons) is at

`http://homepages.vub.ac.be/~phuybrec/ismip.html`

Upload of data is by anonymous ftp at

`ftp://ftp.ulb.ac.be/pub/exchange/fpattyn/incoming`

When you have uploaded a file, please send an email to Frank Pattyn (fpattyn@ulb.ac.be) as your files will only reside on the ftp server for two weeks.

5.3 Upload format

Units to be used:

- Time in years
- distances in metres
- 'volumes' (= areas in our 2-D exeriments) in metres^2
- fluxes in metres^2 per year

The general output file name on upload is to begin with information in the format `NNNn_Ee_Mm_Aa`, with the following components:

- `NNN` = three letter character for name, made up by the first character of the first name followed by the first two characters of the last name, e.g. `FPA` = Frank Pattyn

- `n` = model number (`n` = 1 if you have only one model, or 2, 3, ... if you have more than one model)
- `E` = experiment number (1, 2, or 3)
- `e` = either 'a' or 'b', depending on the two sliding settings (C and m) according to Table 3.
- `M` = control character denoting the grid size
- `m` = grid mode (1, 2 or 3, according to the three different cases, i.e., '1' for 100 equally spaced grid points, '2' for 1000 equally spaced grid points, or '3' for own choice. The latter should be documented in a separate file)
- `A` = control character denoting the value of A (flow parameter)
- `a` = value in the range 1 to 16, corresponding to the value of the flow parameter, as given in table 4 for experiments 1 and 2, or tables 5 and 6 for experiment 3.

Example: model 2 by Frank used on experiment 1b, with grid mode 2 and the third value of flow parameter A (i.e., 'run no. 3' in table 4) would be uploaded using file names beginning with `FPA2_1b_M2_A3`

Further extensions and the format of data in the files depend on the type of information that is contained in them and partly on the type of model used:

- For the data calculated every 50 years in experiments 1 and 3, use filenames of the form `NNNn_Ee_Mm_Aa_t`. This file is a tabulated or space delimited text file that contains either five, twelve or sixteen columns (the number of lines depends on the number of plotted time sequences, every 50 years). Five columns are required if your model uses equation (2), in which case he columns are:
  t x_g Volume $h(0,t)$ $h(x_g,t)$
  For models that do not use (2) but ensure that the flotation condition is satisfied at the grounding line instead of differentiating it, you will need either thirteen or seventeen columns. If your model does not include a shelf, use thirteen columns, with the format
  t x_g Volume $h(0,t)$ $h(x_g,t)$ x_1 $h(x_1,t)$, $b(x_1)$ $q(x_1,t)$ x_2 $h(x_2,t)$ $b(x_2)$ $q(x_2,t)$
  Remember that the sign convention for b is positive down. Here, x_1 and x_2 are the positions of two grid points immediately to the grounded side of the grounding line. One of these can be the grounding line itself, say $x_1 = x_g$, but even in that case x_1 must be entered in its own column despite the fact that it simply repeats the values given in the x_g column. For models that resolve the shelf, you will require seventeen columns, with the format
  t x_g Volume $h(0,t)$ $h(x_g,t)$ x_1 $h(x_1,t)$ $b(x_1)$ $q(x_1,t)$ x_2 $h(x_2,t)$ $b(x_2)$ $q(x_2,t)$ x_3 $h(x_3,t)$ $b(x_3)$ $q(x_3,t)$
  where x_3 is the position of a grid point immediately on the floating side of the grounding line (and x_3 *must not* coincide with the grounding line).

- For the steady state results in experiments 1, 2 and 3, use file names of the form NNNn_Ee_Mm_Aa_ss. These files are tabulated or space delimited text files with two columns (the number of lines depends on the number of grid points used and starts from the ice divide), of the form
  $x_i \quad h(x_i, t_f)$
  where x_i are grid point positions. If your model *does not* assume hyrdostatic equilibrium, then these files should be tabulated or space delimited text files with three columns, of the form
  $x_i \quad h(x_i, t_f) \quad l(x_i, t_f)$
  where $l(x_i, t)$ is the depth of lower boundary of the ice below sea level at grid point position x_i

- Provide the steady state grounding line positions and times at which they were attained separately, using file names NNNn_Ee_Mm_Aa_f, and provide tabulated or space delimited text files with two entries each, of the form
  $x_g(t_f) \quad t_f$
  corresponding to final grounding line position and time needed to reach that position.

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Parameter	physical meaning
ρ_i	ice density
ρ_w	water density
g	gravitational acceleration
n	exponent in Glen's law
A	Glen's law coefficient
C	bed friction parameter
m	bed friction exponent, so $\tau_b = C\|u\|^{m-1}$
a	accumulation rate
x	horizontal coordinate
x_g	grounding line position
h	ice thickness
b	depth of the bed below sea level (positive downward!)
q	ice flux
u	ice velocity

Table 1: List of parameters prescribed for the experiments, as well as other symbols.

Parameter	value
ρ_i	900 kg m^{-3}
ρ_w	1000 kg m^{-3}
g	9.8 m s^{-2}
n	3
a	0.3 m year^{-1}

Table 2: List of parameter values used in experiments 1 and 2.

Parameter	experiment 1a/2a	experiment 1b/2b
m	1/3	1
C	7.624×10^6 Pa m$^{-1/3}$ $s^{1/3}$	7.2082×10^{10} Pa m^{-1} s

Table 3: Sliding parameterizations. With the chosen value of C, a basal shear stress of 80 kPa corresponds to a sliding velocity of about 35 m a^{-1}.

step no.	A (s^{-1} Pa^{-3})
1	4.6416×10^{-24}
2	2.1544×10^{-24}
3	10^{-24}
4	4.6416×10^{-25}
5	2.1544×10^{-25}
6	10^{-25}
7	4.6416×10^{-26}
8	2.1544×10^{-26}
9	10^{-26}

Table 4: Values of $\bar{A}$ to be used in experiment 1.

step no.	$\bar{A}$ (s^{-1} Pa^{-3})	time interval (years)
1	3×10^{-25}	3×10^4
2	2.5×10^{-25}	1.5×10^4
3	2×10^{-25}	1.5×10^4
4	1.5×10^{-25}	1.5×10^4
5	1×10^{-25}	1.5×10^4
6	5×10^{-26}	3×10^4
7	2.5×10^{-26}	3×10^4
8	5×10^{-26}	1.5×104
9	1×10^{-25}	1.5×10^4
10	1.5×10^{-25}	3×10^4
11	2×10^{-25}	3×10^4
12	2.5×10^{-25}	3×10^4
13	3×10^{-25}	1.5×10^4

Table 5: Values of $\bar{A}$ and time intervals to be used in experiment 3a.

step no.	$\bar{A}$ (s^{-1} Pa^{-3})	time interval (years)
1	1.6×10^{-24}	3×10^4
2	1.4×10^{-24}	1.5×10^4
3	1.2×10^{-24}	1.5×10^4
4	10^{-24}	1.5×10^4
5	8×10^{-25}	1.5×10^4
6	6×10^{-25}	1.5×10^4
7	4×10^{-25}	1.5×10^4
8	2×10^{-25}	3×10^4
9	4×10^{-25}	1.5×10^4
10	6×10^{-25}	1.5×10^4
11	8×10^{-25}	1.5×10^4
12	10×10^{-25}	1.5×10^4
13	12×10^{-25}	1.5×10^4
14	14×10^{-25}	3×10^4
16	16×10^{-25}	1.5×10^4

Table 6: Values of $\bar{A}$ and time intervals to be used in experiment 3b.